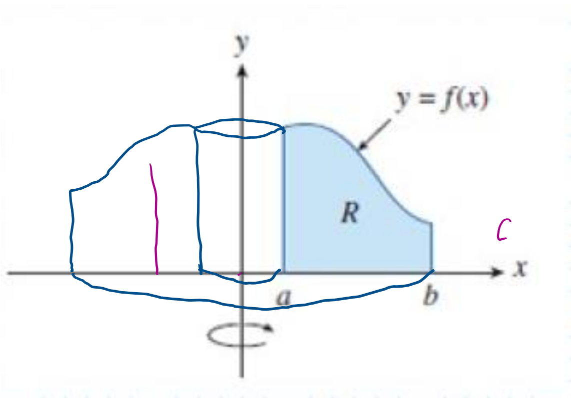
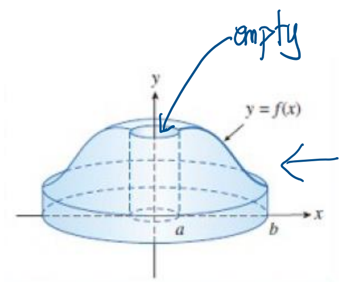
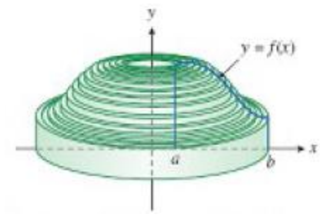
Find Volume Using Cylindrical Shells

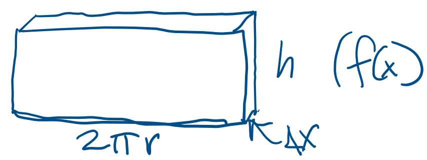
Recall: Sometimes volumes are impractical (if not impossible) to find using disk or washer method.







$V_{CYL\ SLICE}$ = $2\pi r \cdot h \cdot \Delta x$



Cylindrical Shell Volume Formula: $V = 2\pi \int_a^b radius \cdot height\ dx$ or dy

ex. Find the volume of solid obtained by rotating about *y*-axis the region between $y = x$ and $y = x^2$ using cylindrical shells.

$$V = 2\pi \int_0^1 \overset{r}{x}\overset{h}{(x - x^2)}\,dx$$

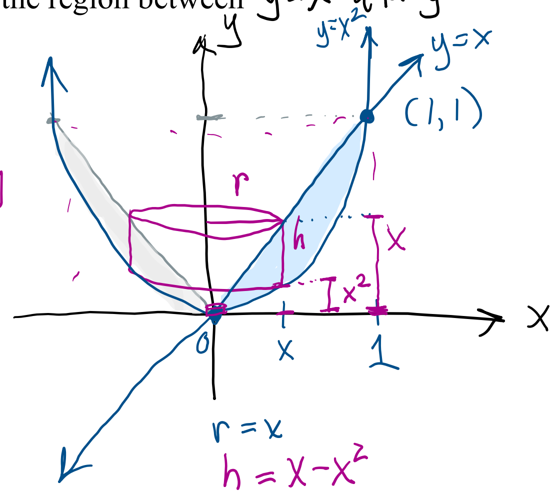
$$= 2\pi \int_0^1 (x^2 - x^3)\,dx$$

$$= 2\pi \left(\frac{x^3}{3} - \frac{x^4}{4}\right)\Big|_0^1$$

$$= 2\pi\left(\frac{1}{3}\cdot\frac{4}{4} - \frac{1}{4}\cdot\frac{3}{3} - 0\right)$$

$$= \cancel{2}\pi\left(\frac{4-3}{\cancel{12}\,6}\right) = \boxed{\frac{\pi}{6}}$$

Do: finish finding volume



r = x
h = x − x²

ex. Find the volume of region bound by $y = x - x^2$ and $y=0$, rotated about $x=2$.

REGION:

$y = -x^2 + x$

y-int: $(0,0)$

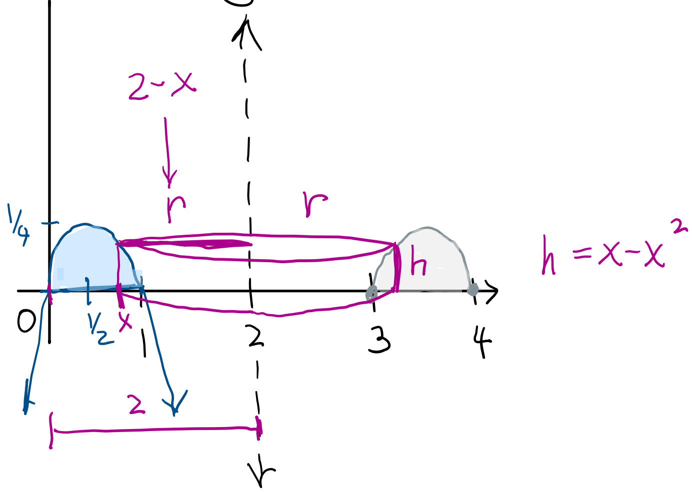
x-int: $x - x^2 = 0$

$x(1-x) = 0$

$x = 0$ or $x = 1$

vertex: $x = -\frac{b}{2a} = \frac{-1}{2(-1)} = \frac{1}{2}$

$y_{x=\frac{1}{2}}: -\left(\frac{1}{2}\right)^2 + \frac{1}{2} = \frac{1}{4}$



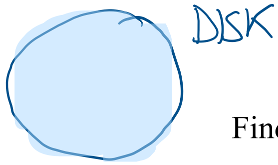
$V = 2\pi \int_0^1 (2-x)(x-x^2)\,dx$ Do: finish finding volume

$= 2\pi \int_0^1 (2x \underbrace{- 2x^2 - x^2} + x^3)\,dx$

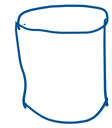
$= 2\pi \int_0^1 (2x - 3x^2 + x^3)\,dx$

$= 2\pi \left(x^2 - x^3 + \frac{x^4}{4} \right) \Big|_0^1$

$= 2\pi \left(\cancel{1} - \cancel{1} + \frac{1}{4} - 0 \right) = 2\pi \cdot \frac{1}{4} = \boxed{\frac{\pi}{2}}$



$A_{DISK} = \pi r^2$



r = consistent

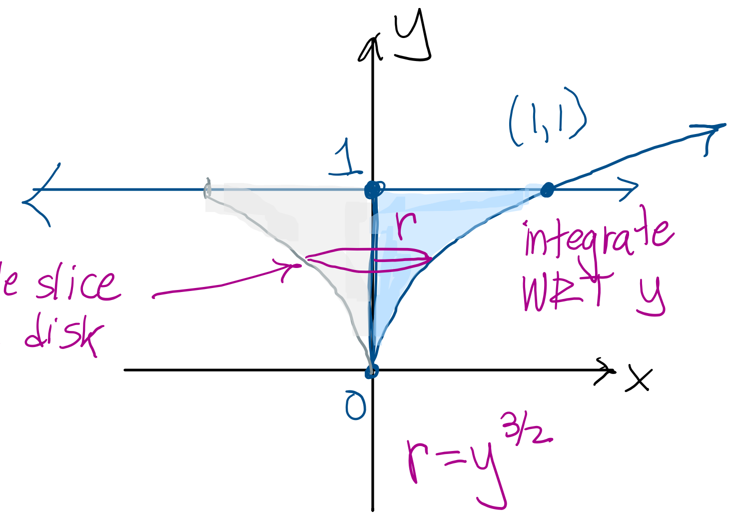
Recall: Disk Method $V = \pi \int_a^b radius^2 \, dx$ or dy

ex. Find the volume of region bound by $y = x^{2/3}$, $y = 1$, $x = 0$, rotate about y-axis.

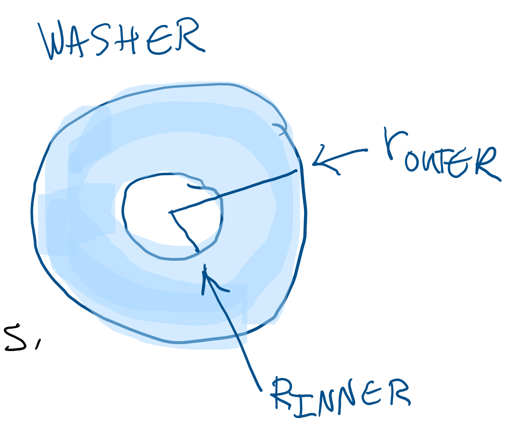
x	$x^{2/3}$
0	0
1	1

Do: solve for x
$x = y^{3/2}$

$$V = \pi \int_0^1 \left(y^{3/2}\right)^2 dy$$



$$V = \pi \int_0^1 y^3 \, dy$$
$$= \pi \left.\frac{y^4}{4}\right|_0^1$$
$$= \pi \left(\frac{1}{4} - 0\right) = \boxed{\frac{\pi}{4}}$$



Recall: Washer Method $V = \pi \int_a^b \left[\left(r_{outer}\right)^2 - \left(r_{inner}\right)^2\right] dx$ or dy

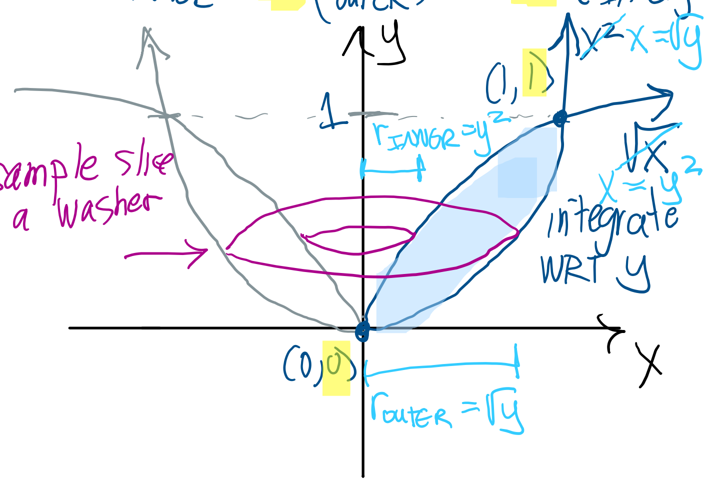
ex. Find the volume of region *using Washer Method* bound by

$y = \sqrt{x}$ and $y = x^2$, rotate about y-axis.

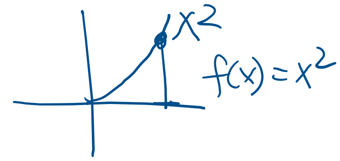
intersect @ $x = 0$, $x = 1$

$x = y^2$ $\qquad$ $x = \sqrt{y}$

$A_{WASHER} = \pi (r_{OUTER})^2 - \pi (r_{INNER})^2$



$$V = \pi \int_0^1 \left(\sqrt{y}^{\,2} - \left(y^2\right)^2\right) dy$$
$$= \pi \int_0^1 \left(y - y^4\right) dy$$
$$= \pi \left.\left(\frac{y^2}{2} - \frac{y^5}{5}\right)\right|_0^1$$
$$= \pi \left(\frac{1}{2} \cdot \frac{5}{5} - \frac{1}{5} \cdot \frac{2}{2} - 0\right) = \boxed{\frac{3\pi}{10}}$$



ex. Find the volume of region *using cylindrical shells* bound by $y=\sqrt{x}$, $y=x^2$, rotated about y-axis

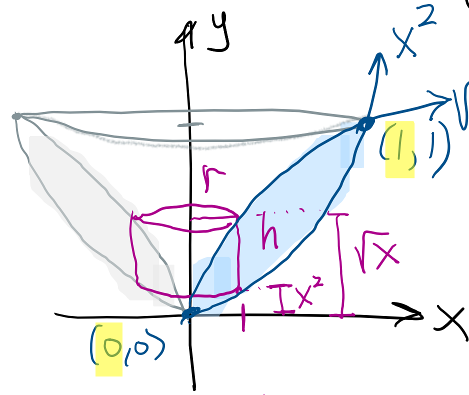
$$V = 2\pi\int_0^1 \Big[\underbrace{x}_{r}\big(\underbrace{\sqrt{x}-x^2}_{h}\big)\Big]\,dx \qquad \sqrt{x}=x^{1/2}$$

$$= 2\pi\int_0^1 \left(x^{3/2} - x^3\right)dx$$

$$= 2\pi\left(\frac{2}{5}x^{5/2} - \frac{x^4}{4}\right)\Bigg|_0^1$$

$$= 2\pi\left(\frac{2}{5}\cdot\frac{4}{4} - \frac{1}{4}\cdot\frac{5}{5} - 0\right)$$

$$= \cancel{2}\pi\left(\frac{8-5}{\cancel{20}\,10}\right) = \boxed{\frac{3\pi}{10}}$$



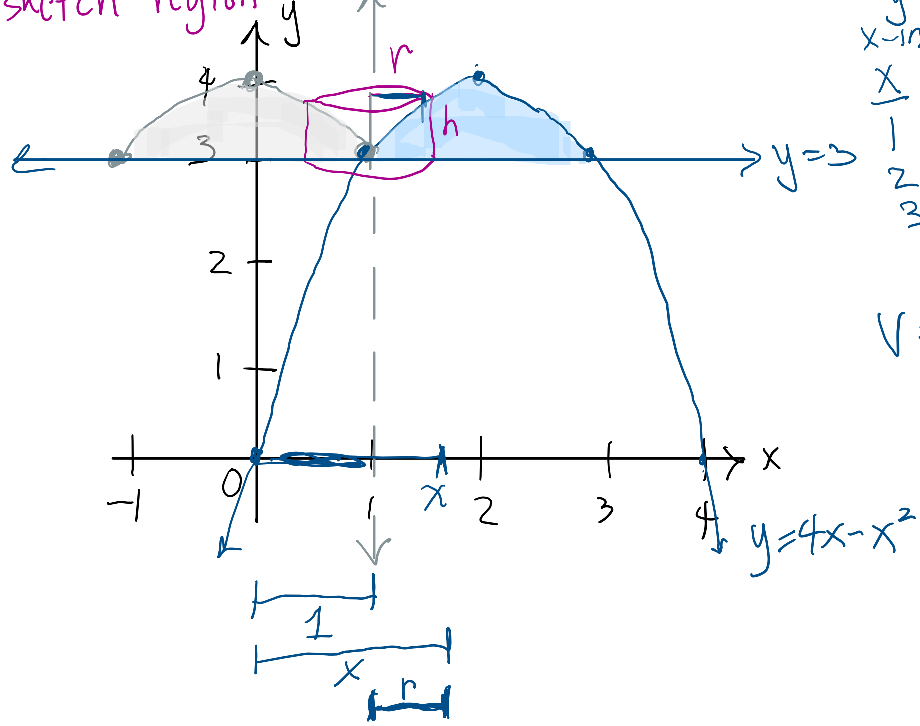
$r = x$

$h = \sqrt{x} - x^2$

Recall: How to Set Up Integral When Region is Revolved Around Something Other than *x*- or *y*-axis:

ex. Find the volume of region *using cylindrical shells* bound by $y=4x-x^2$, $y=3$, rotate about $x=1$.

Do: sketch region



y-int @ 0

x-int @ 0, 4

x	y
1	3
2	4 ← vertex
3	3

$$V = 2\pi\int_1^3 \underbrace{(x-1)}_{r}\underbrace{(4x-x^2-3)}_{h}\,dx$$

$\vdots$

$$= \boxed{\frac{8\pi}{3}}$$